Integrating, substituting value of b, and reducing,

$$V_2=\frac{(d-r)[3a^2r^2-(d-r)^2(2ah-h^2)]}{3r\sqrt{(2ah-h^2)}}$$

$$\left[\pi-2\sin^{-1}\left(\frac{r(a-h)}{\sqrt{[a^2r^2-(d-r)^2(2ah-h^2)]}}\right)\right]$$

$$+\frac{\pi r^2h(3a-h)}{3(2a-h)}-\tfrac{4}{3}(d-r(a-h)\sqrt{(2dr-d^2)}$$

$$+\frac{4a^3r^2}{3(2ah-h^2)}\tan^{-1}\left(\frac{(a-h)(d-r)}{a\sqrt{(2rd-d^2)}}\right)-\frac{2r^2(a-h)\ (2a^2+2ah-h^2)}{3(2ah-h^2)}\sin^{-1}\left(\frac{d-r}{r}\right).$$

If $d=2r$, $V_2=\dfrac{2\pi r^2h(3a-h)}{3(2a-h)}$.

If $d=2r$, $a=h$, $V_2=\frac{4}{3}\pi r^2h$. Total volume$=V+V_2$.

Also solved by *G. W. GREENWOOD*.

162. Proposed by J. E. SANDERS, Hackney, O.

Solve the differential equations

$$(a)\ x\frac{dy}{dx}-y=x\sqrt{(x^2+y^2)},\quad (b)\ \cos x\frac{dy}{dx}+y=1-\sin x.$$

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.; E. L. SHERWOOD, Professor of Mathematics, Shady Side Academy, Pittsburg, Pa.; M. E. GRABER, Graduate Student, Heidelberg University, Tiffin, O.; and the PROPOSER.

(a) $xdy-ydx=x[x^2+y^2]dx$. Let $y=vx$. $\therefore dy=vdx+xdv$.

$$\therefore dx=\frac{dv}{\sqrt{[1+v^2]}}.\quad \therefore x+A=\log\{v+\sqrt{[1+v^2]}\}.$$

$$\therefore x+A=\log\left(\frac{y+\sqrt{[x^2+y^2]}}{x}\right).$$

(b) $\cos xdy+ydx=[1-\sin x]dx$. Let $y=v[1-\sin x]$.

$\therefore dy=[1-\sin x]dv-v\cos xdx$. $\therefore dx=\cos xdv-v\sin xdx=d[v\cos x]$.

$$\therefore x+A=v\cos x=\frac{y\cos x}{1-\sin x}.\quad \therefore y\cos x=[x+A][1-\sin x].$$

Solved in the same way by *HOMER B. HIGLEY*, *LON C. WALKER*, and *J. SCHEFFER*.